Time 2hrs INDIAN STATISTICAL INSTITUE – CHENNAi CENTRE

5/Sept/2016 Mid-Term : Probability (M-STAT I YEAR) Max: 30

Attempt all questions.

I Let $\{E_n\}$ be a sequence of events from $(\Omega, \mathcal{B}, P)$.

(a) Define $\limsup_{n \to \infty} E_n$.

(b) Suppose $P(E_n) = \frac{1}{2^n}$, $n = 1, 2, \ldots + \infty$. Compute $P(\limsup E_n)$.

[2 + 3]

II At a party n men take off their hats. The hats are mixed up and each man randomly selects one hat. We say a match occurs if a man selects his own hat. Let X be the number of men that select their own hats. Compute $E(X)$ and variance of X.

[2+3]

III A hen wants to cross a one way street, where cars ~~are driven~~ arrive and driven according to a homogeneous Poisson process with intensity λ cars a time unit, all with the same speed. It takes c time units for the hen to cross the road. Assume the hen starts to cross the road immediately when there is a chance to do it without being run over by a car. Compute the expected total waiting and crossing time for the hen.

[5]

(Hint: Number the arriving cars by 1, 2, 3, ... and so on. Denote by N the number of the first car which allows the hen to cross the road safely. So the total waiting and crossing time is $X_1 + X_2 + \cdots + X_{N-1} + c$ when X_i is the interarrival time for the i^{th} car)

IV Examine whether the following statements are true or false. Give reasons in either case.

(a) In a finite Markov chain, there exists at least one state which is not transient.

(b) Let $\Omega = (-\infty, \infty)$. Let $\mathcal{a}_n$ be the smallest σ-field of subsets of Ω generated by $\{[0,1), [1,2) \ldots [n-1, n)\}$

Then $\bigcup_{n=1}^{\infty} \mathcal{a}_n$ is a σ-field.

(c) Consider the following transition probability matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{2}{6} & \frac{3}{6} \\ 0 & 0 & 0 & 1/2 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 3/4 \end{bmatrix}_{6 \times 6}$$

In this Markov chain states ~~2, 3 and 4~~ 2 and 3 are the only transient states. [4+4+4]

Brevity + Neatness . . . . [3].